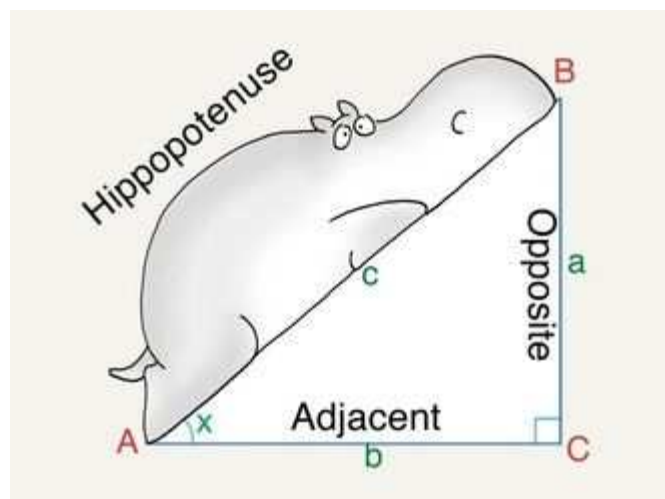
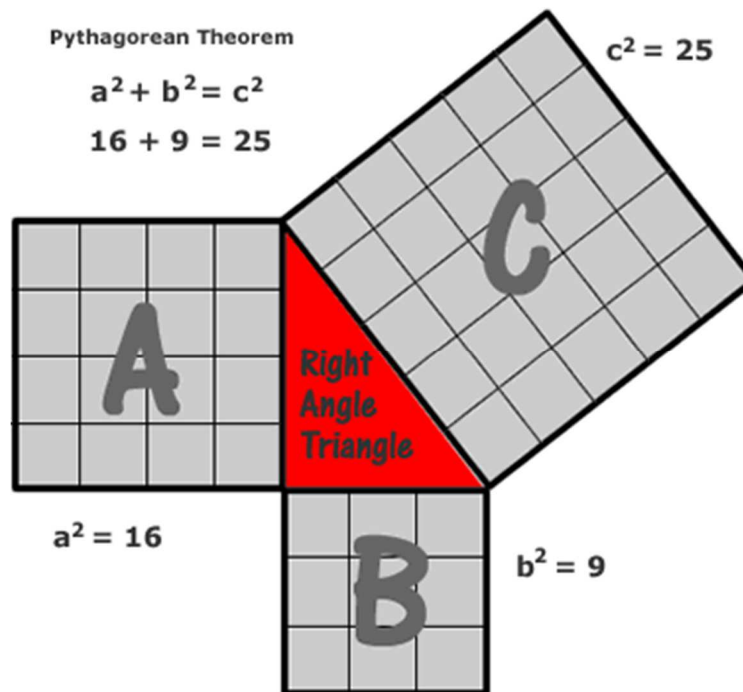


# TRIGONOMETRY

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

$$16 + 9 = 25$$



## Pythagorean Theorem:

**Right Triangle** - a triangle with one \_\_\_\_\_ angle (or  $90^\circ$ ).

**Hypotenuse** - the \_\_\_\_\_ side of a triangle,

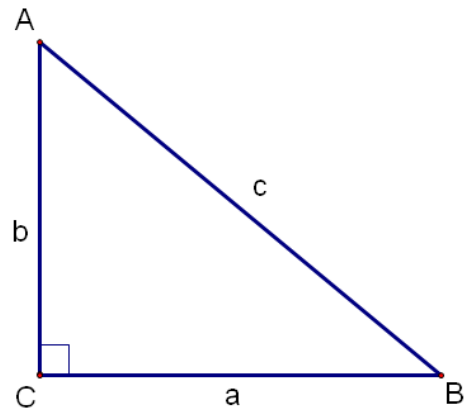
It is always \_\_\_\_\_ the  $90^\circ$  angle.

Example: side \_\_\_\_\_

**Legs** - the two sides that intersect to form a

\_\_\_\_\_ angle.

Example: side \_\_\_\_\_ and side \_\_\_\_\_.



Pythagorean Theorem may be used to determine the \_\_\_\_\_ of the sides in right triangles.

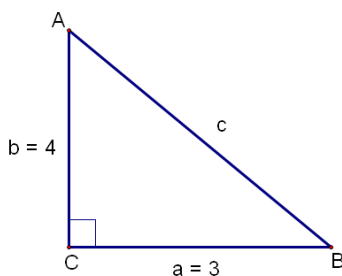
**Pythagorean Theorem** states the sum of the \_\_\_\_\_ of the lengths of the legs is \_\_\_\_\_ to the square of the length of the \_\_\_\_\_.



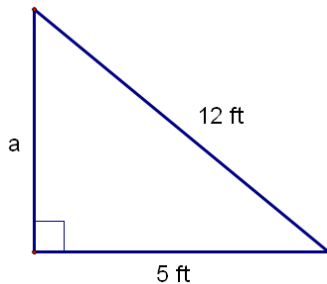
The *longest* side of the right triangle is typically represented by the letter \_\_\_\_\_.

The two *shorter* sides are typically represented by the letters \_\_\_\_\_ and \_\_\_\_\_.

Example 1:  $\triangle ABC$  is a right triangle. If  $\angle C = 90^\circ$ ,  $a = 3$  and  $b = 4$ , find side  $c$ . (5)



Example 2: A ladder is placed against a wall so that a painter can paint the side of the house. The ladder, the wall, and the ground form a right triangle. If the ladder is 12 feet long and is placed 5 feet from the base of the wall, how high does the ladder reach (round to 2 decimal places). (10.91 ft)



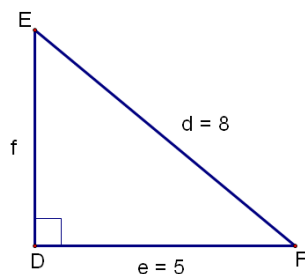
Sometimes, the length of the **hypotenuse** will be given, and you will be required to find the length of one of the **legs**.

Pythagorean Theorem may be used to determine the \_\_\_\_\_ of one of the legs in right triangles.

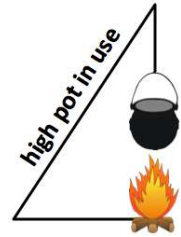
**Pythagorean Theorem** may be rewritten to solve for the length of a missing leg.



Example 3:  $\triangle DEF$  is a right triangle. If  $\angle D = 90^\circ$ ,  $d = 8$  and  $e = 5$ , what is the length of  $f$ . (6.24)

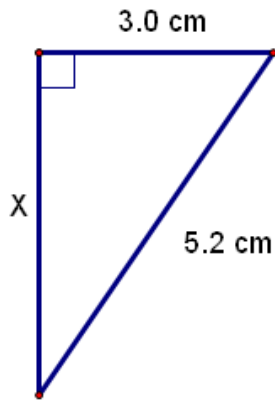


## Pythagorean Practice Questions:

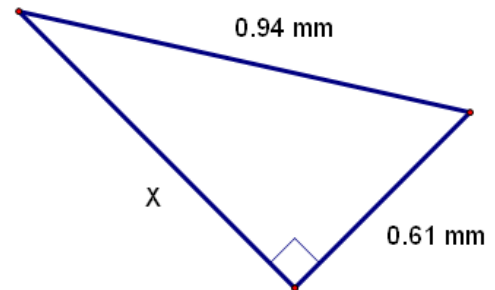


Solve for  $x$ . Round answers to 2 decimal places. Circle final answer. Show your work!

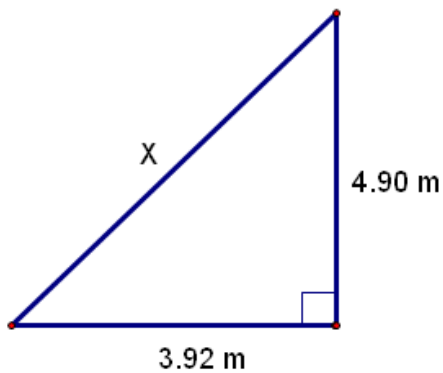
1)



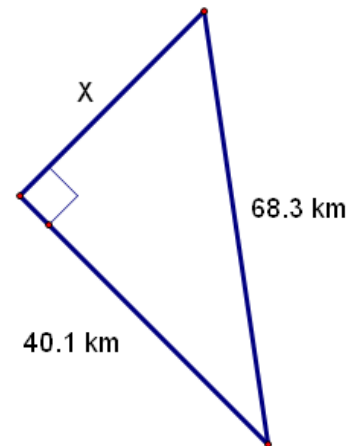
2)



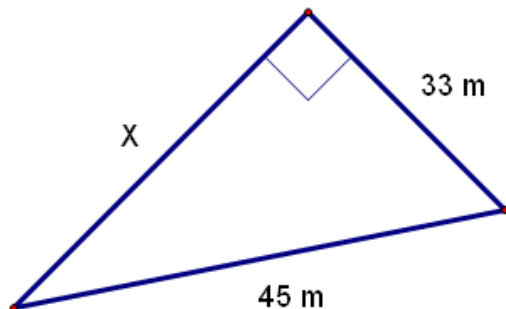
3)



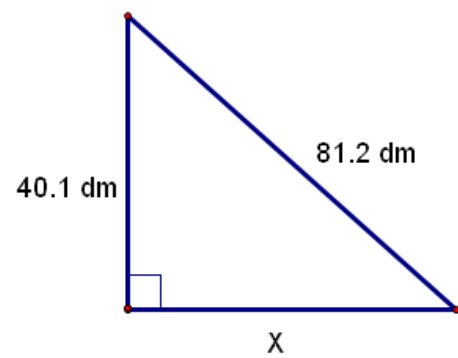
4)



5)



6)



(1. 4.25 cm; 2. 0.72 mm; 3. 6.28 m; 4. 55.2 km; 5. 30.59 m; 6. 70.61 dm)

## The Trigonometric Ratios:

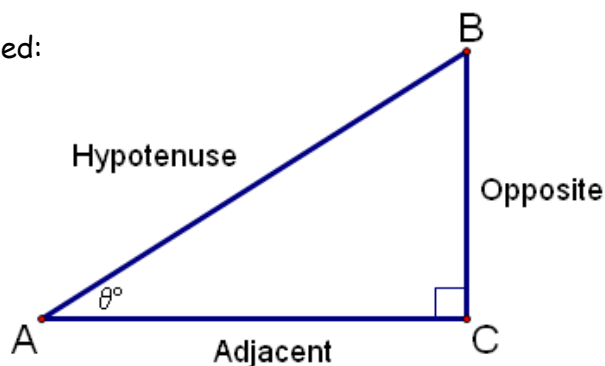
When referring to \_\_\_\_\_ related

to an \_\_\_\_\_, specific words are used:

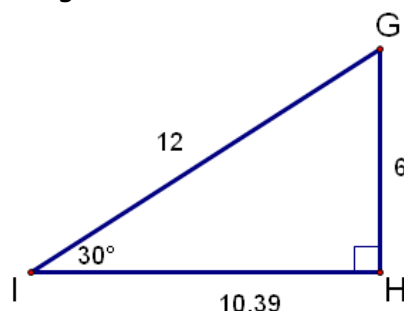
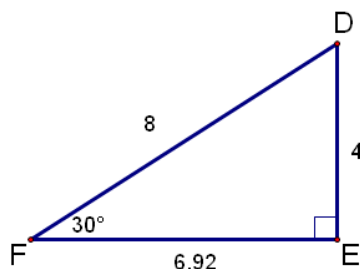
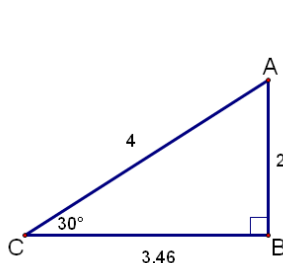
- Side **BC** is said to be \_\_\_\_\_  $\angle A$  or  $\theta$ .

- Side **AC** is said to be \_\_\_\_\_  
(beside or next to)  $\angle A$  or  $\theta$ .

- AB** is the \_\_\_\_\_.



Let's examine some features of triangles. The following three triangles are *similar*.



**SINE:** Notice the **ratio** of  $\frac{\text{length of } \text{side}}{\text{length of}}$  for each triangle:

$$\frac{AB}{AC} = \frac{2}{4} =$$

$$\frac{DE}{DF} =$$

$$\frac{GH}{GI} =$$

Notice that this ratio is always the \_\_\_\_\_.

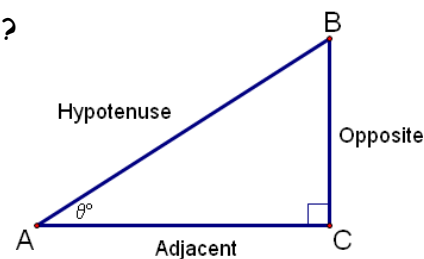
**COSINE:** What about the ratio of  $\frac{\text{length of}}{\text{length of hypotenuse}}$  side ?

Are these also equal? \_\_\_\_\_

$$\frac{BC}{AC} = \frac{3.46}{4} =$$

$$\frac{EF}{DF} =$$

$$\frac{HI}{GI} =$$



**TANGENT**: And, what about the ratio of  $\frac{\text{length of opposite side}}{\text{length of adjacent side}}$  ?

$$\frac{AB}{BC} = \frac{2}{3.46} = 0.58$$

$$\frac{DE}{EF} = \frac{4}{6.92} = 0.58$$

$$\frac{GH}{HI} = \frac{\quad}{\quad} = 0.58$$

These 3 ratios have been given special names.

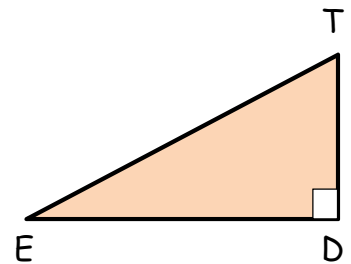
**THE SINE RATIO** - the **ratio** of the length of the side \_\_\_\_\_ a given angle to the length of the \_\_\_\_\_. (Abbreviation = **sin**)

$$\sin \theta = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

OR  $\sin \theta = \frac{\quad}{\quad}$

OR

Example:  $\sin E =$



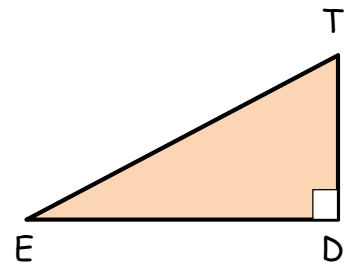
**THE COSINE RATIO** - the **ratio** of the length of the side \_\_\_\_\_ to a given angle to the length of the \_\_\_\_\_. (Abbreviation = **cos**)

$$\cos \theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

OR  $\cos \theta = \frac{\quad}{\quad}$

OR

Example:  $\cos E =$



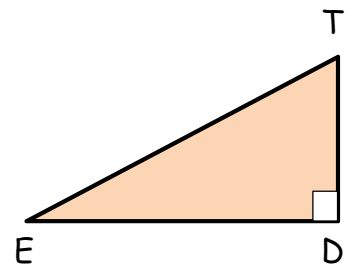
**THE TANGENT RATIO** - the **ratio** of the length of the side \_\_\_\_\_ a given angle to the length of the \_\_\_\_\_ side. (Abbreviation = **tan**)

$$\tan \theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

OR  $\tan \theta = \frac{\quad}{\quad}$

OR

Example:  $\tan E =$

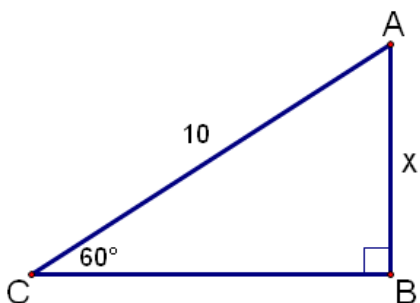


**SOH CAH TOA**

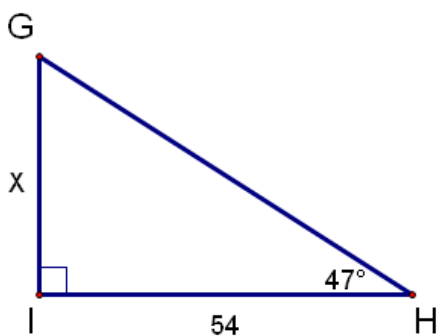
How do I remember these 3 ratios? **SOH CAH TOA**

**Make sure your calculator is in DEGREES! Round answers (if needed) to 2 decimal places.**

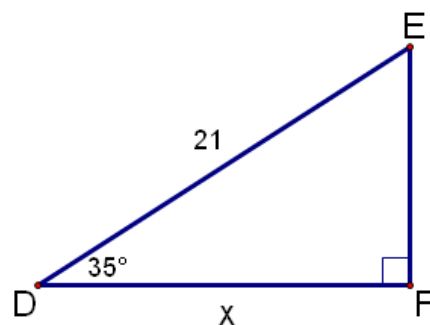
Example 1: Solve for  $x$  in the following triangle using the \_\_\_\_\_ ratio. (8.66)



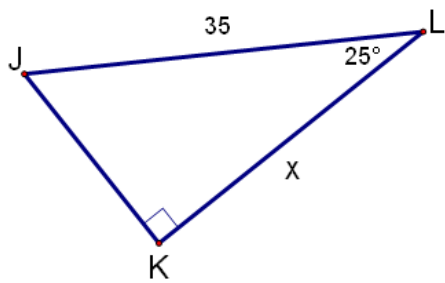
Example 2: Solve for  $x$  in the following triangle using the \_\_\_\_\_ ratio. (57.91)



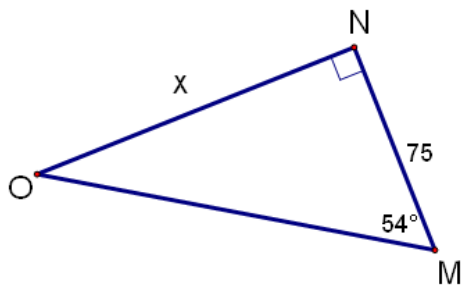
Example 3: Solve for  $x$  in the following triangle using the \_\_\_\_\_ ratio. (17.20)



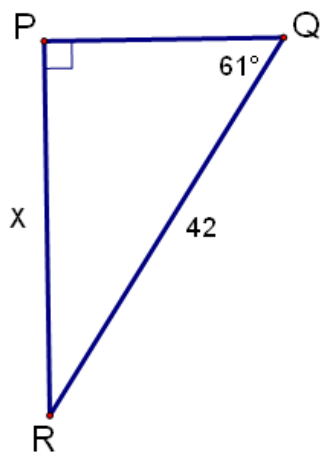
Example 4: Solve for  $x$  in the following triangle. (31.72)



Example 5: Solve for  $x$  in the following triangle. (103.23)



Example 6: Solve for  $x$  in the following triangle. (36.73)





## Assignment #1 - Trigonometric Ratios:

(1) 2.5m; 2) 9.19m; 3) 3.25cm 4) 14.35cm; 5) 1.84dm; 6) 119.20mm; 7) 0.97m; 8) 1.69km)

Solve for  $x$  in the following triangles. Round all answers to 2 decimal places. Show ALL work!

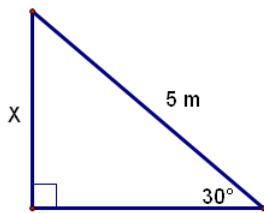
$$\sin \theta = \underline{\hspace{2cm}}$$

$$\cos \theta = \underline{\hspace{2cm}}$$

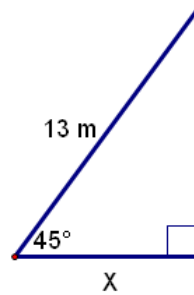
$$\tan \theta = \underline{\hspace{2cm}}$$

( $\theta$  = angle)

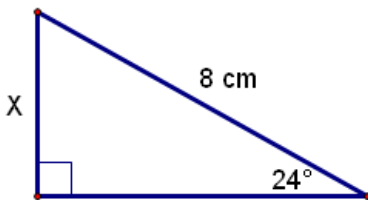
1)



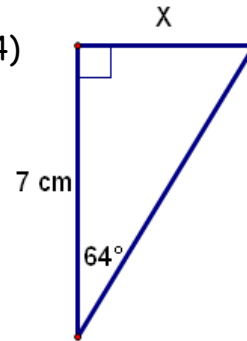
2)



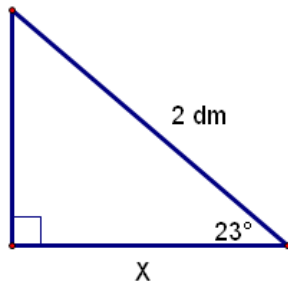
3)



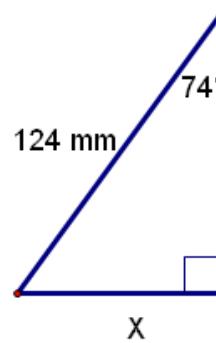
4)



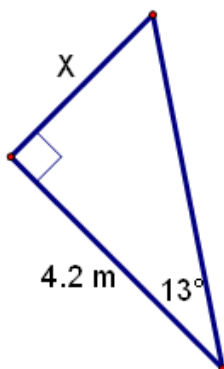
5)



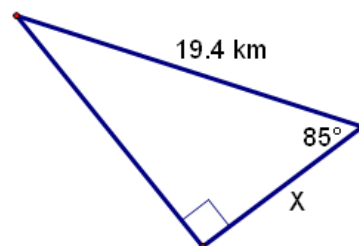
6)



7)

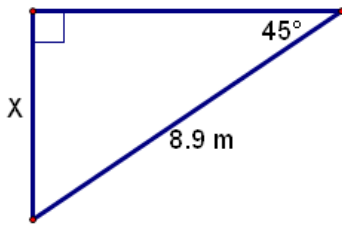


8)

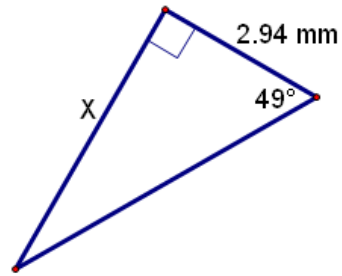


extra practice

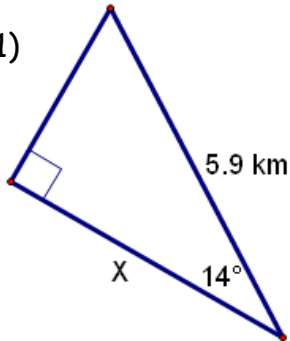
9)



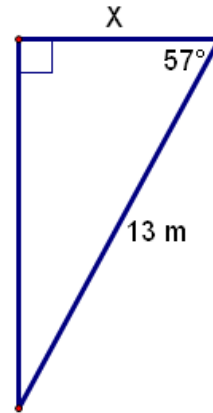
10)



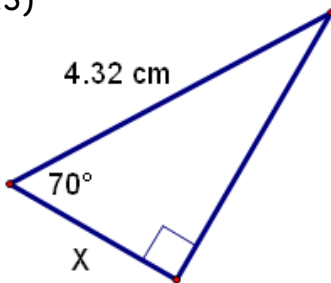
11)



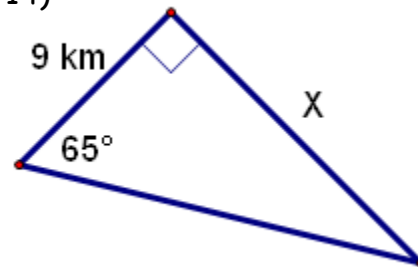
12)



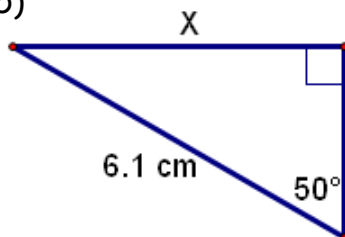
13)



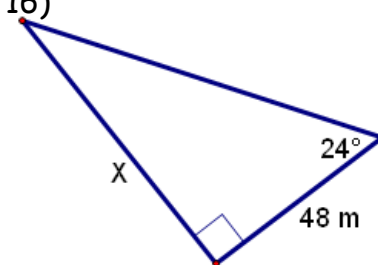
14)



15)



16)



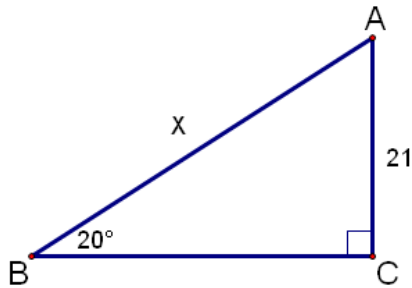
Answers: (9) 4.45 m; (10) 3.38 mm; (11) 5.72 km; (12) 7.08 m; (13) 1.48 cm;; (14) 19.30 km; (15) 4.67 cm; (16) 21.37 m)

## The Trigonometric Ratios - Part 2:

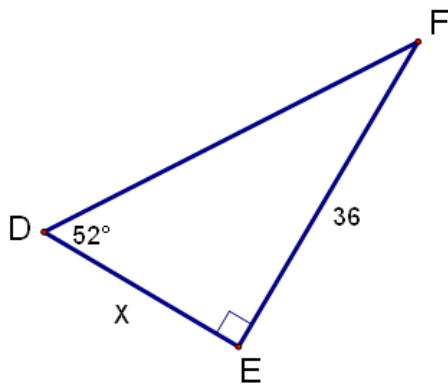
In all of the previous examples, when we were solving for  $x$ , it was in the \_\_\_\_\_ (top) of the trigonometric ratio.

Today we will learn how to solve for  $x$  when it is in the \_\_\_\_\_ (bottom) of the trigonometric ratio.

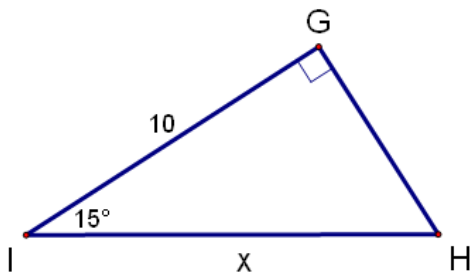
Example 1: Solve for  $x$  in the following triangle. (61.40)



Example 2: Solve for  $x$  in the following triangle. (28.13)



Example 3: Solve for  $x$  in the following triangle. (10.35)



## Assignment #2 - Trigonometric Ratios Part 2:

1) 7.07m; 2) 6.93km; 3) 5.87cm; 4) 20.96km; 5) 12.36mm; 6) 9.23dm; 7) 4.08hm; 8) 5.94dm

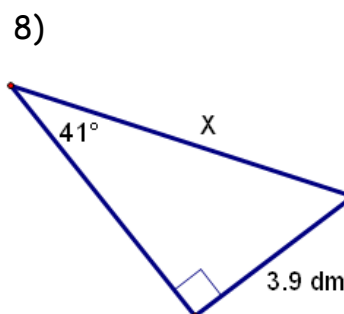
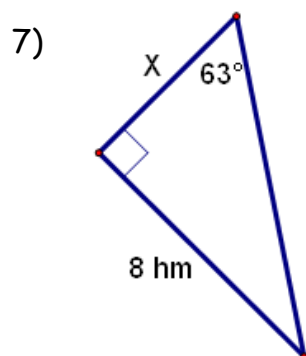
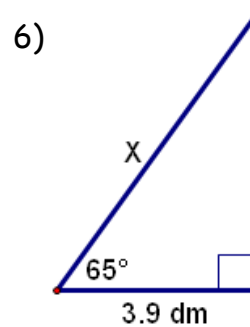
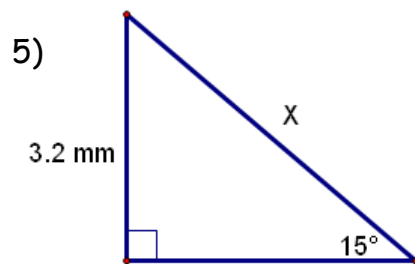
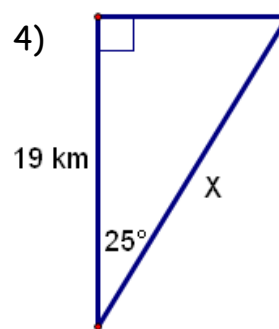
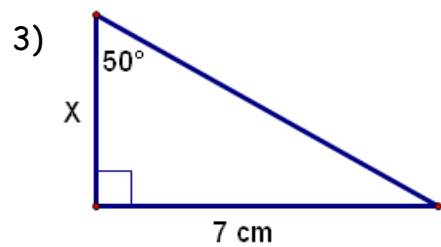
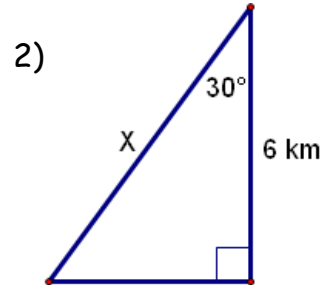
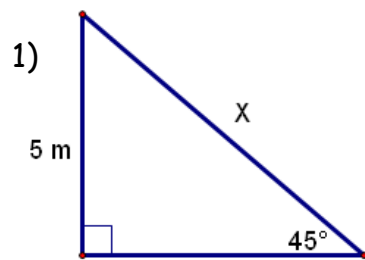
Solve for  $x$  in the following triangles. Round all answers to 2 decimal places.

$$\sin \theta = \underline{\hspace{2cm}}$$

$$\cos \theta = \underline{\hspace{2cm}}$$

$$\tan \theta = \underline{\hspace{2cm}}$$

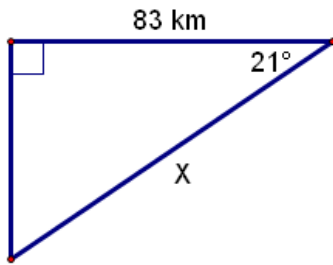
( $\theta$  = angle)



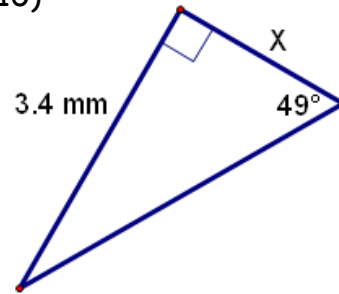
9) 88.9 km; 10) 2.96 mm; 11) 1384.86 m; 12) 232.19 cm; 13) 141 m ; 14) 84.08 dm; 15)  $23^\circ$ ; 16)  $66^\circ$

#9-14 extra practice

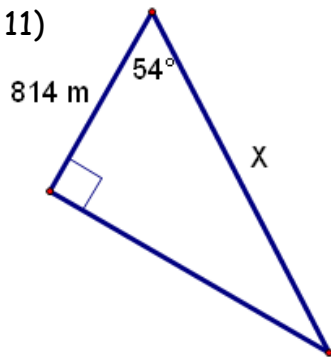
9)



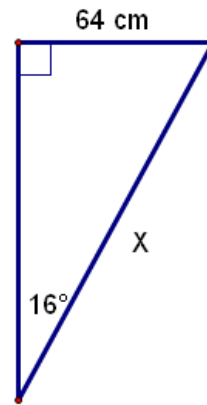
10)



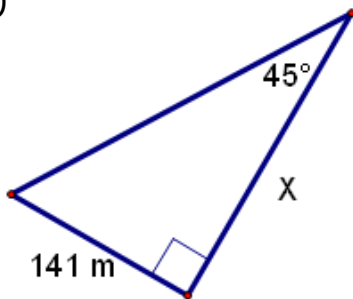
11)



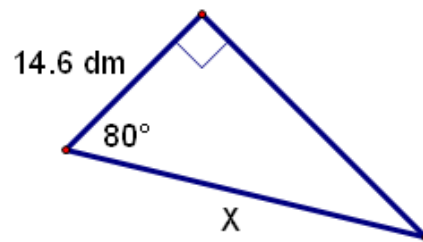
12)



13)

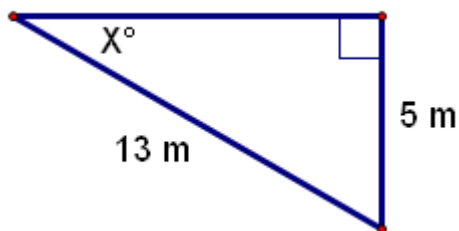


14)

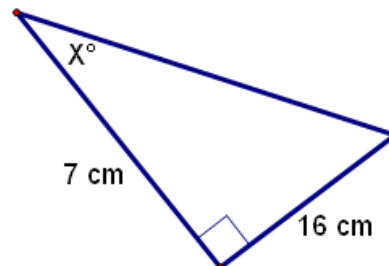


**Try these.** Hint: To find the missing angle, fill in trig ratios as per normal. Use inverse of ratio to find angle.

15) \*



16) \*



## The Trigonometric Ratios -Angles (round to nearest degree)

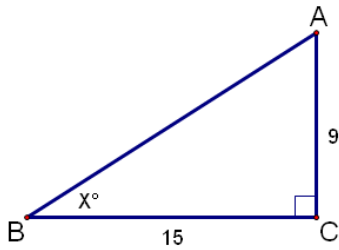
Sometimes, you may need to find the measure of an \_\_\_\_\_.

To find an angle you must use the "\_\_\_\_\_" trigonometric functions on your calculator.

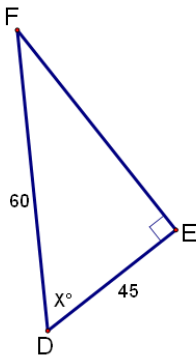
$$\theta = \sin^{-1}\left(\frac{\quad}{\quad}\right) \quad \theta = \cos^{-1}\left(\frac{\quad}{\quad}\right) \quad \theta = \tan^{-1}\left(\frac{\quad}{\quad}\right)$$

To do this, you will need to use the "*2<sup>nd</sup>*" or "*SHIFT*" button on your calculator.

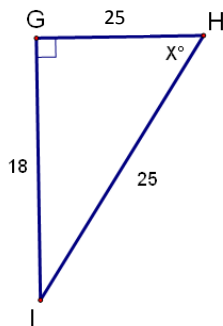
Example 1: Solve for x in the following triangle. (31°)



Example 2: Solve for x in the following triangle. (41°)



Example 3: Solve for x in the following triangle. (46°)



### Assignment #3 - Trigonometric Ratios - Angles:

Answers: (1)  $49^\circ$ ; (2)  $55^\circ$ ; (3)  $58^\circ$ ; (4)  $53^\circ$ ; (5)  $21^\circ$ ; (6)  $41^\circ$ ; (7)  $64^\circ$ ; (8)  $41^\circ$

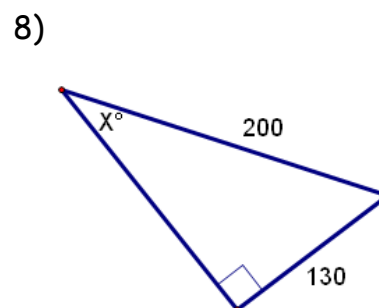
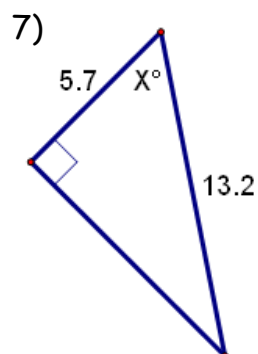
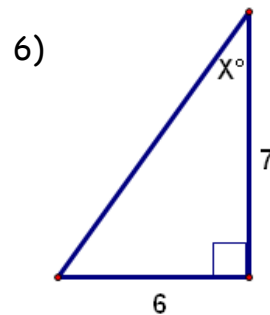
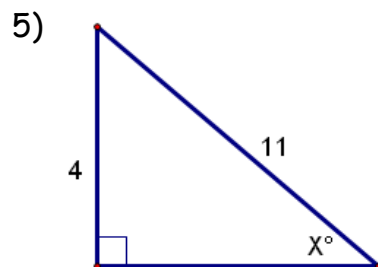
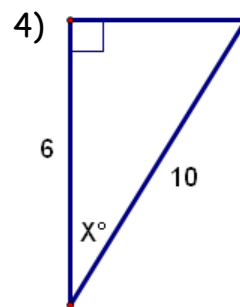
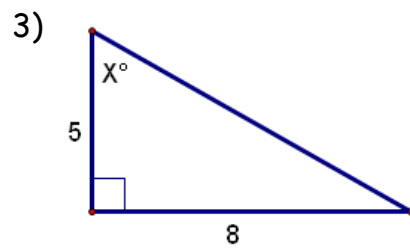
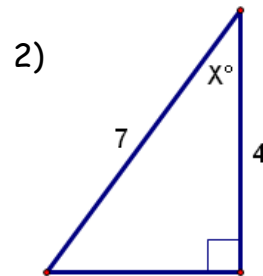
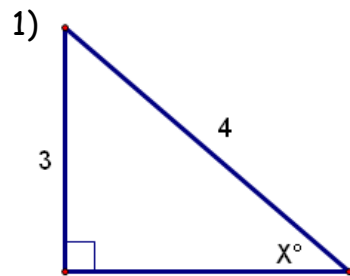
Solve for  $x$  in the following triangles. Round all answers to the nearest degree.

$$\sin \theta = \underline{\hspace{2cm}}$$

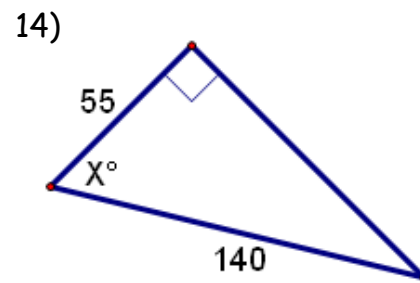
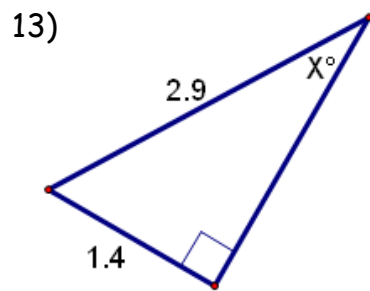
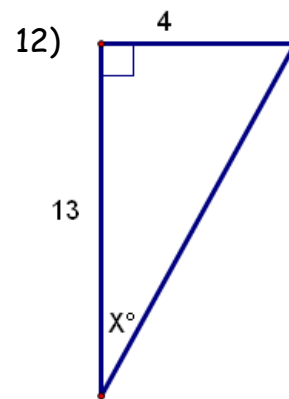
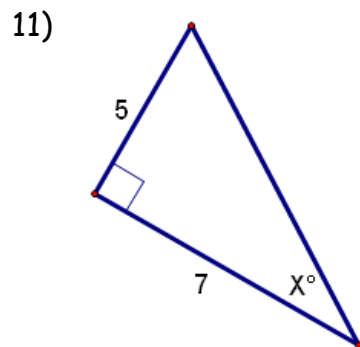
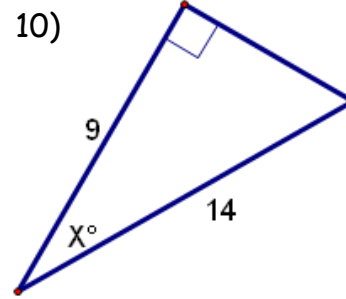
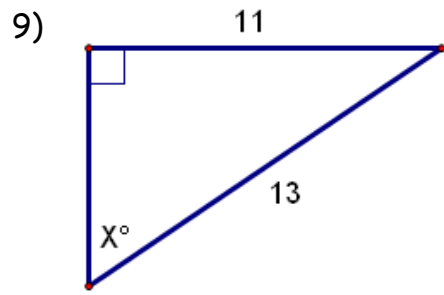
$$\cos \theta = \underline{\hspace{2cm}}$$

$$\tan \theta = \underline{\hspace{2cm}}$$

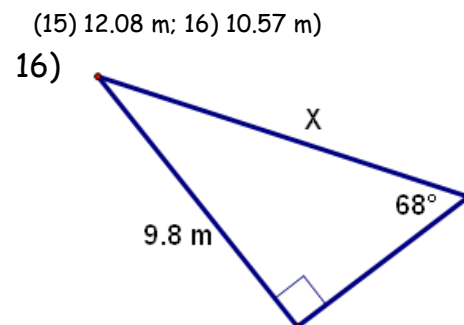
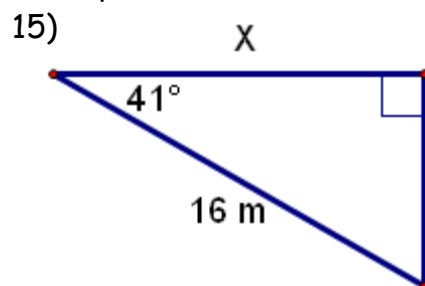
( $\theta$  = angle)



9-14 Extra practice Round to nearest degree. (9)  $58^\circ$ ; 10)  $50^\circ$ ; 11)  $36^\circ$ ; 12)  $17^\circ$ ; 13)  $29^\circ$ ; 14)  $67^\circ$



Try these. Round to 2 decimal places.



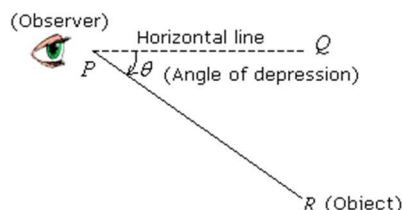
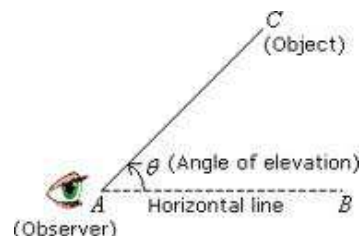


## Trigonometry - Word Problems:

Remember:

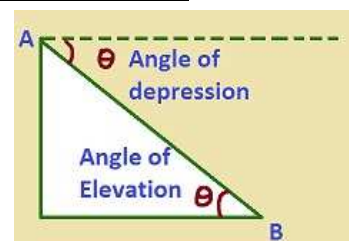
- When solving word problems, make sure to \_\_\_\_\_ the problem carefully.
- Making a \_\_\_\_\_ with labels is always the first step!

**Angle of Elevation** - the angle formed between the horizontal and the line of sight while looking \_\_\_\_\_.



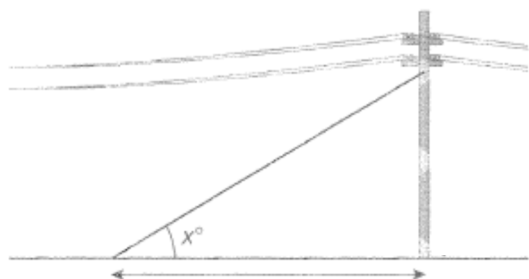
**Angle of Depression** - the angle formed between the horizontal and the line of sight while looking \_\_\_\_\_.

**Note:** For any given problem, the angle of elevation will have the \_\_\_\_\_ value as the angle of depression.



Example 1: A wire, 69.8 m in length, is attached to the top of a tower. The wire makes an angle of  $63^\circ$  with the ground. How high is the tower, rounded to 2 decimal places? (62.19 m)

Example 2: How far from the base of a pole (rounded to 2 decimal places) must a 6.2 m long guy wire be attached if the angle of elevation is  $65^\circ$ ? (2.62 m)



Example 3: A guy wire 8.5 m long is attached 5.7 m from the base of a pole. Find the angle (*rounded to nearest degree*) between the ground and a guy wire. ( $48^\circ$ )

Example 4: Find the angle of depression (*rounded to nearest degree*) to a point 10.1 m downhill if the horizontal distance is 6.9 m. ( $47^\circ$ )

### Assignment #4 - Trigonometry Word Problems:

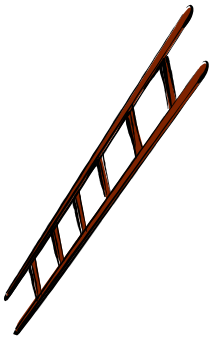
*Round sides to 2 decimal places; angles to nearest degree.*

1. Draw and label diagram.
2. Write trig ratio you will use.
3. Substitute numbers into ratio and simplify.
4. Write answer in a sentence including units.

- 1) The length of the kite string is 86 m. The angle that the string makes with the ground is  $36^\circ$ . Calculate the vertical height of the kite. (50.55m)



- 2) An 8 m ladder is leaned against a wall, with the base of the ladder 1.2 m from the wall. Find the angle between the ladder and the ground. ( $81^\circ$ )



- 3) If the angle of elevation to the top of a tree is  $30^\circ$  from a point 12 m from its base, how tall is the tree? (6.93 m)



- 4) Lucy must have a wheelchair ramp built to her front porch. The porch is 1.9 m above ground level and the steepest angle of elevation allowed by the building code is  $6^\circ$ .
- a) What is the shortest ramp that Lucy can have installed? (18.2 m)
  - b) About how many metres from the base of the porch must the ramp start? (18.08 m)
- 5) The ramp down to an underground garage is 12 m long. If the floor of the garage is 3 m below the level of the street, find the angle of elevation of the ramp. ( $15^\circ$ )
- 6) A crime scene investigator (CSI) is investigating a bullet hole in the side of a building. The hole is 2.4 m above the floor and entered the wall at an angle of  $83^\circ$ . If the suspect was lying inside the building and on the ground when he took the shot, about how far from the wall was he? (0.29 m)

## Trigonometry Problems - Two Triangles:

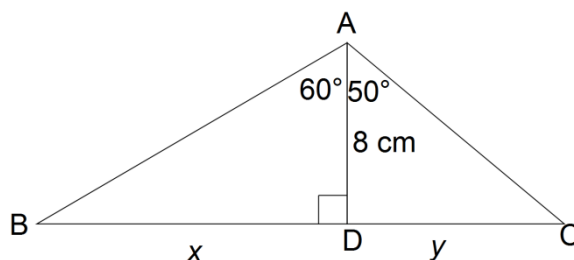
When solving a problem with two triangles, try to identify a common \_\_\_\_\_ between the two triangles, or two sides or two angles that you can \_\_\_\_\_.

These types of problems will involve \_\_\_\_\_ one step to answer the question.

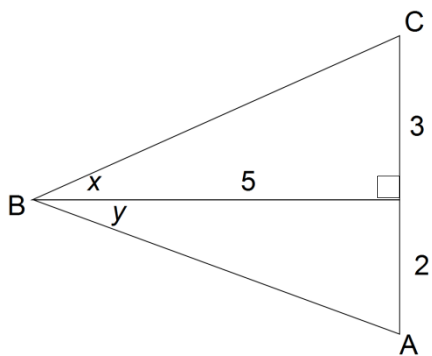
Decide what angles and sides you need to find and **label with letters like X, Y, Z.**

**Remember to not round off until the final answer.** Leave the sides and angles truncated to 4 decimal places. For the final answer, the side can be rounded to 2 decimal places and the angle to the nearest degree.

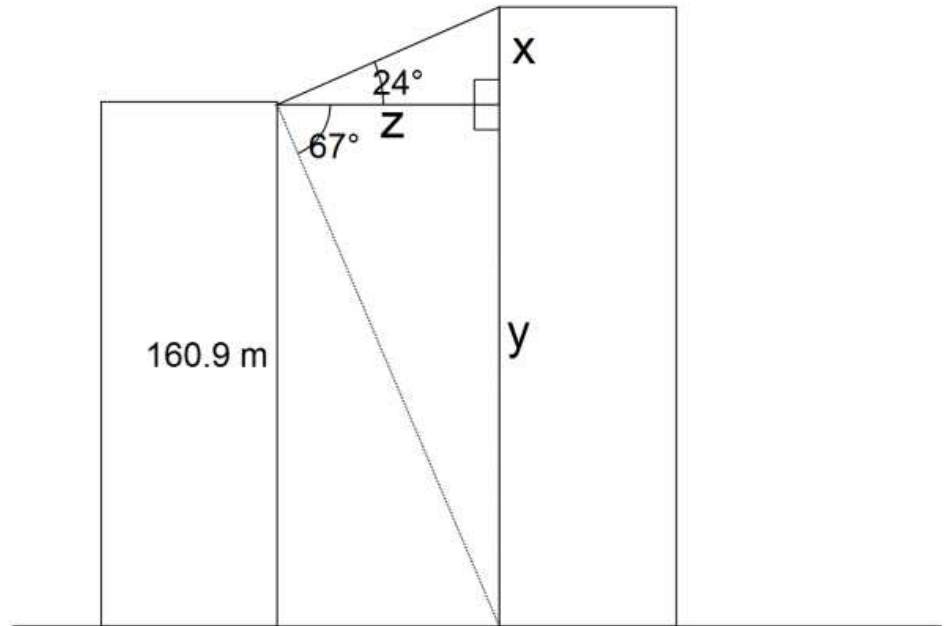
Example 1: Calculate the length of BC. (23.39 cm)



Example 2: Calculate the measure of  $\angle ABC$ , rounded to nearest degree. ( $53^\circ$ )



Example 3: Two office towers are a certain distance apart. The shortest building has a height of 160.9 m. The angle of depression from the top of the shorter tower to the base of the taller tower is  $67^\circ$ . The angle of the elevation from the same point to the top of the taller tower is  $24^\circ$ . What is the height of the taller tower? (191.31 m)



## Assignment #5 - Word Problems with Two Triangles

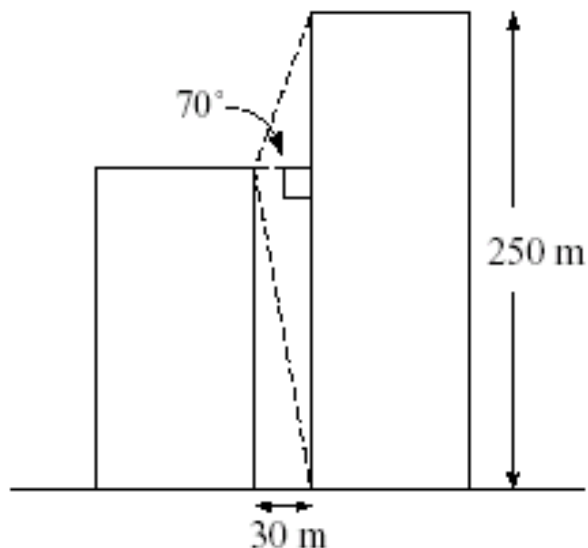
Remember to first sketch a diagram if diagram not given.

For each step:

1. Write trig ratio you will use.
  2. Substitute numbers into ratio and simplify.
  3. Write answer truncated to 4 decimal places.
- Write final answer in a sentence rounded to 2 decimal places for a side, to nearest degree for an angle.

- 1) Two office towers are 30 m apart. From the top of the shorter tower, the angle of elevation to the top of the other tower, which is 250 m high, is  $70^\circ$ .

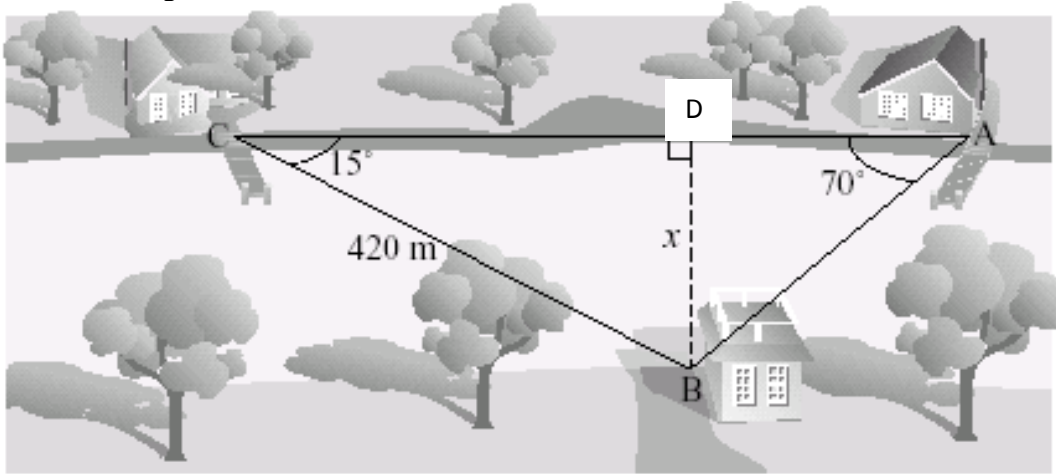
- a) How much taller is the taller tower, truncated to 4 decimal places? (82.4243 m)



- b) Determine the height of the shorter tower, truncated to 4 decimal places? (167.5757 m)  
(hint: not using trig)

- c) Determine the angle of depression (rounded to nearest degree) to the base of the taller tower from the top of the shorter tower. ( $10^\circ$ )

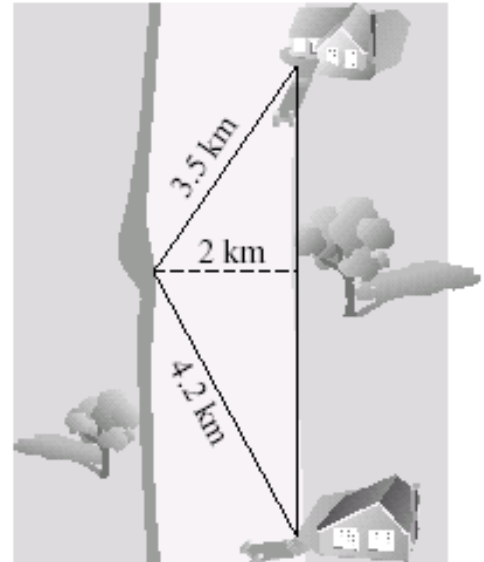
- 2) Two cabins, A and C are located a distance apart on the bank of a river. On the other side of the river from the two cabins is a boathouse, B. It is 420 m from cabin C to the boathouse and the angle at cabin C between the boathouse and cabin A is  $15^\circ$ . From cabin A, the angle between cabin C and the boathouse is  $70^\circ$ .



- a) Determine  $x$ , truncated to 4 decimal places. (108.7039 m)
- b) Label the distance from C to D as  $y$ . Find  $y$ , truncated to 4 decimal places. (405.6888m)
- c) Label the distance from point D to A as  $z$ . Find  $z$ , truncated to 4 decimal places. (39.5649 m)
- d) How far apart are the cabins (A to C), rounded to 2 decimal places? (445.25 m)



- 3) From a point on the west bank of a river 2 km wide, two speedboats leave for their respective cabins on the east side of the river. The distance to the closer cabin is 3.5 km and the distance to the further cabin is 4.2 km. What is the measure of the angle (to the nearest degree) between the boats' paths? ( $116.71^\circ$ )



- 4) From the top of a 100 m tower, a fire ranger spots two fires. One fire is due east of the tower at an angle of depression of  $16^\circ$ . The other is due west and has an angle of depression of  $23^\circ$ . Calculate the distance between the fires. (583.93 m)

